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Flame localization inside axis-symmetric cylindrical and spherical porous media burners

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Abstract

It is shown analytically and by means of numerical simulation that flame front localization inside axis-symmetric cylindrical and spherical porous media burners (hereinafter PMB) is not defined only by fuel flow rate and system parameters, but also depends on combustion ignition coordinate. In a certain parameter range, combustion front does not move significantly from the place of its initial ignition. It is shown that heat losses at the outer boundary of PMB determine the specified peculiarities, with fuel flow rate, heat content of the mixture and the PMB external radius being the governing parameters. © 1998 Elsevier Science Ltd. All rights reserved.

Nomenclature

A, B, C variables
b dimensionless complex in (7)
b' length dimensional complex in (13)
c heat capacity at constant pressure
d_0 diameter of porous carcass grain
D gas diffusivity
E energy of chemical reaction activation in Arrhenius function
G_0 gas mass flow rate
L cylinder length
m carcass porosity
Q the reaction heat release
r radial coordinate
r_1 radius of combustion front location
R universal gas constant
T temperature of the carcass
T_{ad} maximum temperature at gas phase combustion
T_0 ambient temperature
T_1 combustion temperature in one-temperature model
u_g local filtration speed
$W(y, T)$ chemical reaction rate
y concentration of deficit reactant
z pre-exponential of Arrhenius function.

Greek symbols
α volumetric heat transfer coefficient
$\delta(x)$ delta-function
ΔT temperature increase relative to T_0
ε carcass surface emissivity
λ thermal conductivity
ρ density
σ Stefan–Boltzmann constant.

Subscripts
g gas phase
s solid phase
2 outer radius of PMB.

1. Introduction

Gas combustion inside inert porous media or filtration combustion attracts the interest of researchers due to the numerous technical applications of the given process—utilization (oxidation) of hazardous gases, burning of lean combustible mixtures in superadiabatic combustion regime, catalyst regeneration, etc. [1–4]. Application of porous media burners (PMBs) to heat up metals, ceramics, glass and other materials have been studied recently.

Cylindrical axis-symmetrical configuration is one of the possible configurations of porous media burners at which the fuel is supplied into the centre of a ceramic tube and filtered from the centre to the outer surface.

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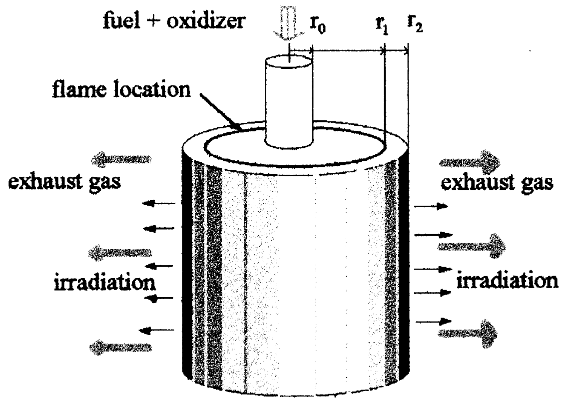

Fig. 1. Sketch of cylindrical axis-symmetric porous media burner (PBM).

Such a configuration provides natural stabilization of combustion front inside the carcass due to filtration speed decrease with the radius growth. Figure 1 shows a flow diagram of cylindrical PMB. A problem of flame localization inside PMB is one of the important questions, since, first, it determines maximum temperature inside the carcass and design of PMB and, second, the localization problem is bound to the combustion front stability. At present the authors are not familiar with any publication concerning combustion localization in cylindrical axis-symmetrical and spherical PMBs. The term 'localization' hereinafter has the particular meaning of stationary flame localization radius.

The present article studies the filtration combustion front localization inside a PMB carcass. In principle, these peculiarities are caused by two factors: (1) radial change of gas filtration speed due to the system geometry, (2) radial dependence of flame heat losses. In this work we have analyzed the system by using one-temperature analytical model and performing one-dimensional two-temperature numerical simulation of combustion front localization. Revealed properties and their practical consequences are discussed.

2. Analytical consideration of combustion localization in cylindrical PMB

To treat the problem of gas filtration combustion in the cylindrical geometry system, we consider a set of balance equations for porous carcass temperature, gas temperature and concentration of deficit reactant:

$$(c\rho)\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\lambda r\frac{\partial T}{\partial r} - \alpha(T-T_g) \quad (1)$$

$$(c\rho)_g\frac{\partial T_g}{\partial t} + (c\rho)_g u_g\frac{\partial T_g}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\lambda_g r\frac{\partial T_g}{\partial r} + \alpha(T-T_g) + QW(y, T_g) \quad (2)$$

$$\frac{\partial y}{\partial t} + u_g\frac{\partial y}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}Dr\frac{\partial y}{\partial r} - W(y, T_g) \quad (3)$$

where T is temperature, y is concentration of deficit reactant, α is volumetric heat transfer coefficient, T_0 is an ambient temperature, Q—reaction heat release, $W(y, T)$—chemical reaction rate, D—diffusivity. Subscript g applies to gas component, solid phase parameters have no subscripts. When changing over to spherically symmetrical systems, the appropriate change of Laplacian takes place.

To consider the problem analytically, change to one-temperature approximation at which temperatures of gas T_g and solid phase T are to be assumed equal due to high intensity of inter-phase heat transfer ($\alpha \to \infty$). Energy balance eqns (1), (2) are reduced to one equation.

$$(c\rho + (c\rho)_g)\frac{\partial T}{\partial t} + u_g(c\rho)_g\frac{\partial T}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}(\lambda+\lambda_g)r\frac{\partial T}{\partial r} + QW(y, T). \quad (4)$$

In case of stationary situation in approximation of instantaneous reaction, and taking into account that $\lambda+\lambda_g \cong \lambda$, the problem becomes

$$(c\rho)_g u_g\frac{\partial T}{\partial r} = 1/r\frac{\partial}{\partial r}\lambda r\frac{\partial T}{\partial r} + Q\rho_g u_g\delta(T-T_1) \quad (5)$$

together with boundary conditions (BC)

$$T(0) = T_0, \quad T(r_1) = T_1, \quad -\lambda\left.\frac{\partial T}{\partial r}\right|_{r_2} = \sigma(T_2^4 - T_0^4). \quad (6)$$

By introducing $G_0 = 2\pi rL\rho_g u_g$—mass flow rate, $b = c_g G_0/(2\pi L\lambda)$, and assuming $\lambda \sim$ const, one can derive from (5)

$$\frac{\partial T}{\partial r} = \frac{1}{b}\frac{\partial}{\partial r}r\frac{\partial T}{\partial r} + \frac{Q}{c_g}\delta(T-T_1) \quad (7)$$

where T_1 is combustion temperature.

General solution (7) has the form

$$T = A + Br^b$$

Using BC (6), one can obtain:

$$T = T_0 + (T_1 - T_0)r_1^{-b}r^b, \quad r < r_1 \quad (8)$$

$$T = (T_1 + C) - Cr_1^{-b}r^b, \quad r_1 < r < r_2$$

$$C = \sigma(T_2^4 - T_0^4)\frac{1}{\lambda b}\frac{r_1^b}{r_2^{b-1}}. \quad (9)$$

To determine the unknown T_1, T_2 and r_1, substitute the values of the derivatives calculated according to (8) and (9) into the first integral of (7).

$$T_2 - T_0 = \Delta T_{ad} - \frac{r_2}{b\lambda}\sigma(T_2^4 - T_0^4) \quad (10)$$

$$T_1 - T_0 = \Delta T_{ad} - (\Delta T_{ad} - \Delta T_2)\frac{r_1^b}{r_2^b}. \quad (11)$$

The stated problem has no single solution, since a set of two equations has three unknowns—T_2, T_1 and r_1. To close it, use the equation for the ignition temperature T_{ign} [5] (assuming $T_{ign} = T_1$)

$$T_1 - T_0 \equiv \Delta T_{ign} = \frac{\Delta T_{ad}}{1+\dfrac{\alpha\lambda}{u_g^2(c\rho)_g^2}} + \frac{E}{R}\left[\ln\left(\frac{Qz}{\alpha}\frac{\left(1+\dfrac{\alpha\lambda}{u_g^2(c\rho)_g^2}\right)}{\Delta T_{ad}}\right)\right]^{-1} - T_0 \quad (12)$$

where u_g—local filtration speed of gas, E—activation energy of chemical reaction, R—universal gas constant, z—pre-exponential of Arrhenius function. Equation (10) is isolated and defines boundary temperature $T_2 < T_{ad}$. Equation (11) gives combustion front temperature T_1 and is a monotonic function of (r_1/r_2) convex upwards at parameter values $b > 1$ and downwards at $b < 1$. Equation (12) gives the dependence of ignition temperature on flow rate, adiabatic temperature and other parameters of filtration combustion. As seen from (12), ignition temperature is a monotonically decreasing function of radius, $T_{ign} \sim u_g^2 \sim 1/r_1^2$. Figure 2 shows the qualitative behaviour of the functions (11) and (12).

It follows from the diagram that one- or two-root solution of the set is possible. At higher flow rates, the two roots converge into one and further the set becomes incompatible, which corresponds to the absence of stationary wave localization in PMB.

Combustion wave behavior can be analysed by using the diagram in Fig. 2. Stable combustion at a given radial position can occur only if T_1 (or T_{ign}) defined by (12) does not exceed the corresponding value obtained from the balance equation solution (11). As a result, one can see that if the mixture is ignited at the PMB centre, temperature front moves downstream to the radius corresponding to the first root of the system. If combustibles

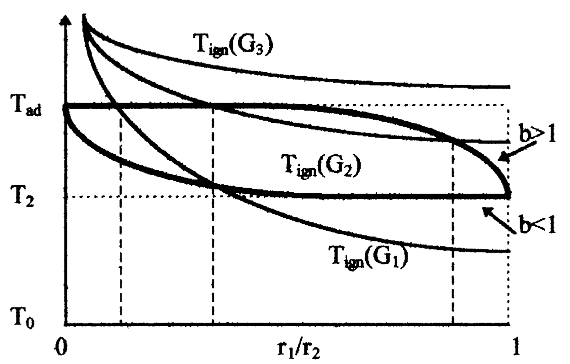

Fig. 2. Diagram of solution for system (10)–(12) at different flow rates and heat content of fuel. Here flow rates G_1, G_2 and G_3 ($G_1 < G_2 < G_3$) correspond to one, two roots or absence of solution of the system, respectively.

are ignited from the outer side of the PMB, (1) combustion front moves to the appropriate localization radius in the case of the existence of one root, or, (2) combustion shifts out of the PMB in the case of two roots or absence of solution. If the combustion is initiated between the first and second roots of the set, the combustion front tends to move to the position of the first root. However, if the combustion is initiated close to the second root, the flame propagation can be strongly retarded or stopped. Thus, first root r_{st1} (corresponding to the smaller radius) gives stable combustion localization and the second one r_{st2} specifies the radius of unstable combustion localization. A situation, when temperature curves determining the system solution (Fig. 2) are coupled to such an extent that the distance between them equates to the value of natural temperature fluctuations in a porous carcass [6], needs special consideration. In this case, flame propagation to radius r_{st1} can slow down or stop and its localization can depend on multiple factors. In this sense, combustion localization becomes indefinite. It should be noted that ignition from the outside is possible in limited interval of flow rates, when there is only one root for localization, while the ignition from inside is possible anyway if there is a root at all.

System (10)–(12) was solved by iterations at fixed outer radius r_2 of the burner. The porous media was a packed bed of $d_0 = 5.6$ mm Al_2O_3 balls. Interphase heat transfer coefficient α was calculated according to [7], heat conductivity of the carcass was estimated as a sum [3] $\lambda = \lambda_{ef} + [32\sigma d_0 m\varepsilon/9(1-m)]T_{ad}^3$, where $\lambda = 1.5$ W/(m K), $m = 0.4$, $\varepsilon = 0.6$.

Typical solutions of system (10)–(12) are presented in Fig. 3. As is seen from the figure, the system may have one, two roots or no solution for flame localization, depending on the fuel flow rate. The area, where localization curve is close to a vertical one, corresponds to the 'indefinite' localization in the sense described above.

Solution of the system demonstrates that localization curves (corresponding to two roots) may surround a closed area on 'flow rate—localization' diagram in Fig. 3 and thus define an area of possible flame localization. External ignition is impossible in this case. This situation may be realized for low calorific content fuels independently of PMB external radius r_2. To combust the leaner fuel, the bigger PMB diameter and smaller packing grain size are necessary.

3. Spherical symmetry of PMB

Similar to cylindrical problem, consider PMB of spherical geometry. One-temperature problem can be written as

$$\frac{\partial T}{\partial r} = \frac{1}{b'}\frac{\partial}{\partial r}r^2\frac{\partial T}{\partial r} + \frac{Q}{c_g}\delta(T-T_1) \quad (13)$$

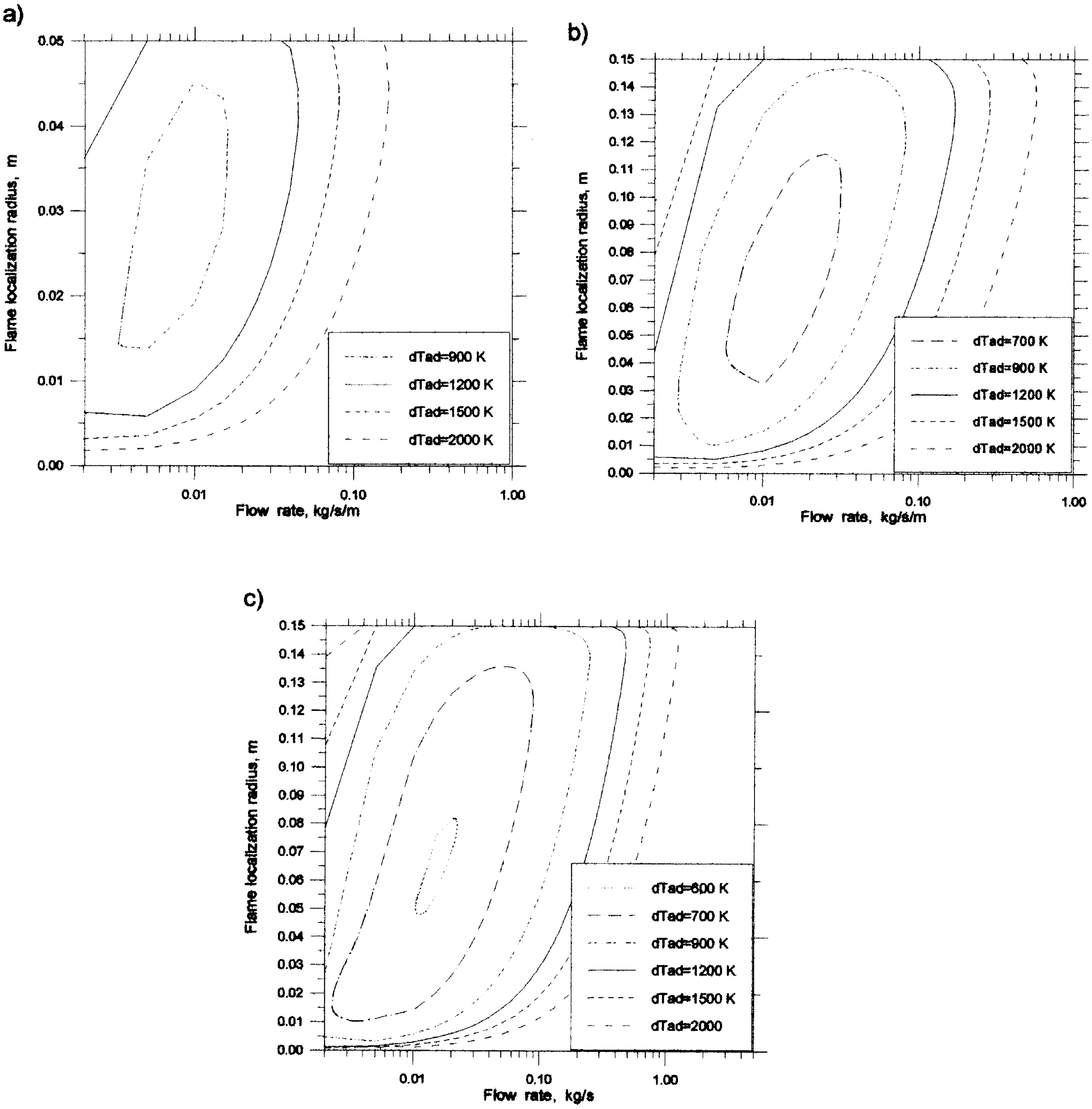

Fig. 3. Solution of the system (10)–(12) for combustion front localization in cylindrical PMB. (a) Outer radius $r_2 = 0.05$ m, $d_0 = 5.6$ mm, (b) $r_2 = 0.15$ m, $d_0 = 5.6$ mm, (c) $r_2 = 0.15$ m, $d_0 = 1$ mm.

with boundary conditions

$$T(0) = T_0, \quad T(r_1) = T_1, \quad -\lambda \frac{\partial T}{\partial r}\bigg|_{r_2} = \sigma(T_2^4 - T_0^4) \tag{14}$$

where we have introduced $G_0 = 4\pi r^2 \rho_g u_g$—mass flow rate, $b' = c_g G_0/(4\pi\lambda)$, $\lambda = \text{const}$.

The solution has the form

$$T = T_0 + (T_1 - T_0)\exp(b'/r_1 - b'/r), \quad r < r_1$$
$$T = (T_1 + C) - C\exp(b'/r_1 - b'/r), \quad r_1 < r < r_2$$
$$C = \sigma(T_2^4 - T_0^4)\frac{r_2^2}{\lambda b'}\exp(b'/r_2 - b'/r_1). \tag{15}$$

To define values T_1, T_2 and r_1, substitute temperature gradients according to (15) into the first integral of (13).

$$T_2 - T_0 = \Delta T_{ad} - \frac{r_2^2}{b'\lambda}\sigma(T_2^4 - T_0^4) \tag{16}$$

$$T_1 - T_0 = \Delta T_{ad} - (\Delta T_{ad} - \Delta T_2)\exp\left(\frac{b'}{r_2} - \frac{b'}{r_1}\right). \tag{17}$$

To close the system, use equation (12). Figure 4 shows solution diagrams for flame localization calculated according to (12), (16)–(17). As follows from a comparison of Figs 3 and 4, ignition from outside is hindered for spherical PMB compared to cylindrical PMB in the sense that a wider layer should be preheated near the surface to get flame propagation inside the burner.

4. Numerical simulation of combustion localization

To test the results of the analytical solution, the numerical simulation was performed within the non-stationary problem statement. We used a two-temperature model (1)–(3), where we neglected the time dependent term for gas temperature and reactant concentration (since $(c\rho)_g \ll (c\rho)_s$), Laplacian terms in (2) and (3), and accepted radiation diffusivity approximation for radiation heat transfer in porous carcass.

Supply of fresh mixture having ambient temperature T_0 to the burner inlet (at $r = r_0$) and irradiation from PMC outer surface (at $r = r_2$) were simulated with boundary conditions

$$T_g|_{r_0} = T_0, \quad \left.\frac{\partial T_g}{\partial r}\right|_{r_2} = 0 \tag{18}$$

for gas and

$$\left.\frac{\partial T}{\partial r}\right|_{r_0} = 0, \quad -\lambda_s \left.\frac{\partial T}{\partial r}\right|_{r_2} = \varepsilon\sigma(T_2^4 - T_0^4) \tag{19}$$

for solid phase.

Contrary to analytical problem statement, where BC were taken as zero, during numerical simulation inner radius r_0 has finite value of 1 cm.

To solve the system of equations (1)–(3) with boundary conditions (18) and (19), iteration procedure was implemented for each time step. Equation (1) was approximated by implicit scheme and was solved by Gaussian two-step elimination. Equations (2) and (3) were integrated explicitly.

The effect of combustion initiation radius on the radius of final flame localization was studied by calculating a series of non-stationary flame trajectories with different radii of combustion ignition. Ignition was simulated by giving high-temperature step ($T = 1200$ K) in the porous body at the initial instant of time. The following ignition radii were taken 0.01, 0.2, 0.4, 0.6, 0.8, 0.9 and 0.93 from the width of the working zone of the PMB. Other ignition radii were taken when necessary.

Physical time corresponding to the computation of the combustion front trajectory was 4 h, which provided good approximation to stationary position.

To test program codes, the adiabatic problem statement was used with boundary condition

$$\left.\frac{\partial T_s}{\partial r}\right|_{r_2} = 0.$$

Under the adiabatic conditions all combustion wave trajectories converged to one and the same stationary radius, regardless of ignition radius, while energy balance in the system being kept with an accuracy of 1–2%.

Major series of computations were performed for methane–air combustion kinetics modelled with overall reaction rate in Arrhenius form $-\partial y/\partial t = yz\exp(-E/RT)$ (where $z = 2.6 \times 10^8\ s^{-1}$, $E = 13 \times 10^4\ J\ mol^{-1}$ [3]) and adiabatic combustion temperatures $\Delta T_{ad} = 1200$, $\Delta T_{ad} = 1500$ and $\Delta T_{ad} = 2000$ K. Mass mixture flow rate G varied in a wide range 0.001–2.5 kg/s. Heat conductivity of the carcass was expressed according to [3] $\lambda = \lambda_{ef} + [32\sigma d_0 m\varepsilon/9(1-m)]T^3$. The values of the other parameters were chosen as follows: $\lambda_{ef} = 1.5$ W m^{-1} K^{-1}, $m = 0.4$, $d_0 = 5.6$ mm, $\varepsilon = 0.6$. Interphase heat transfer coefficient α was calculated according to [7].

Computations confirmed the possibility of existence of two localization radii for the systems under consideration. Figure 5 gives temperature profiles corresponding to two solutions of the problem.

Numerical calculation has confirmed that outer localization radius r_{st2}, if it exists, is unstable and combustion front can stay on the radius long enough only in case of initial ignition on that radius. Otherwise, the front moves towards inner stable localization radius, if the ignition is initiated closer to the centre from r_{st2}, and is blown out from PMB, if the ignition is initiated outside from r_{st2}. As far as small variations of flame location and shape are characteristic for real devices, the flame localization close to r_{st2} is unstable.

Localization radii, data obtained as a result of numerical calculations for PMB of cylindrical geometry as a function of fuel flow rate, its heat content and outer radius are presented in Figs 6 and 7.

The figures show the curves corresponding to analytical solution of the problem at the same parameter values of PMB. As seen, the results of the analytical and numerical solutions coincide qualitatively, which confirms adequacy of analytical model. The differences may be explained by the error of one-temperature approximation and assumption of constant heat conductivity λ of analytical problem consideration and also by nonzero value of the inner PMB radius taken for numerical computation.

The computations have revealed that the wave front velocity slows down in the area of 'indifferent' local-

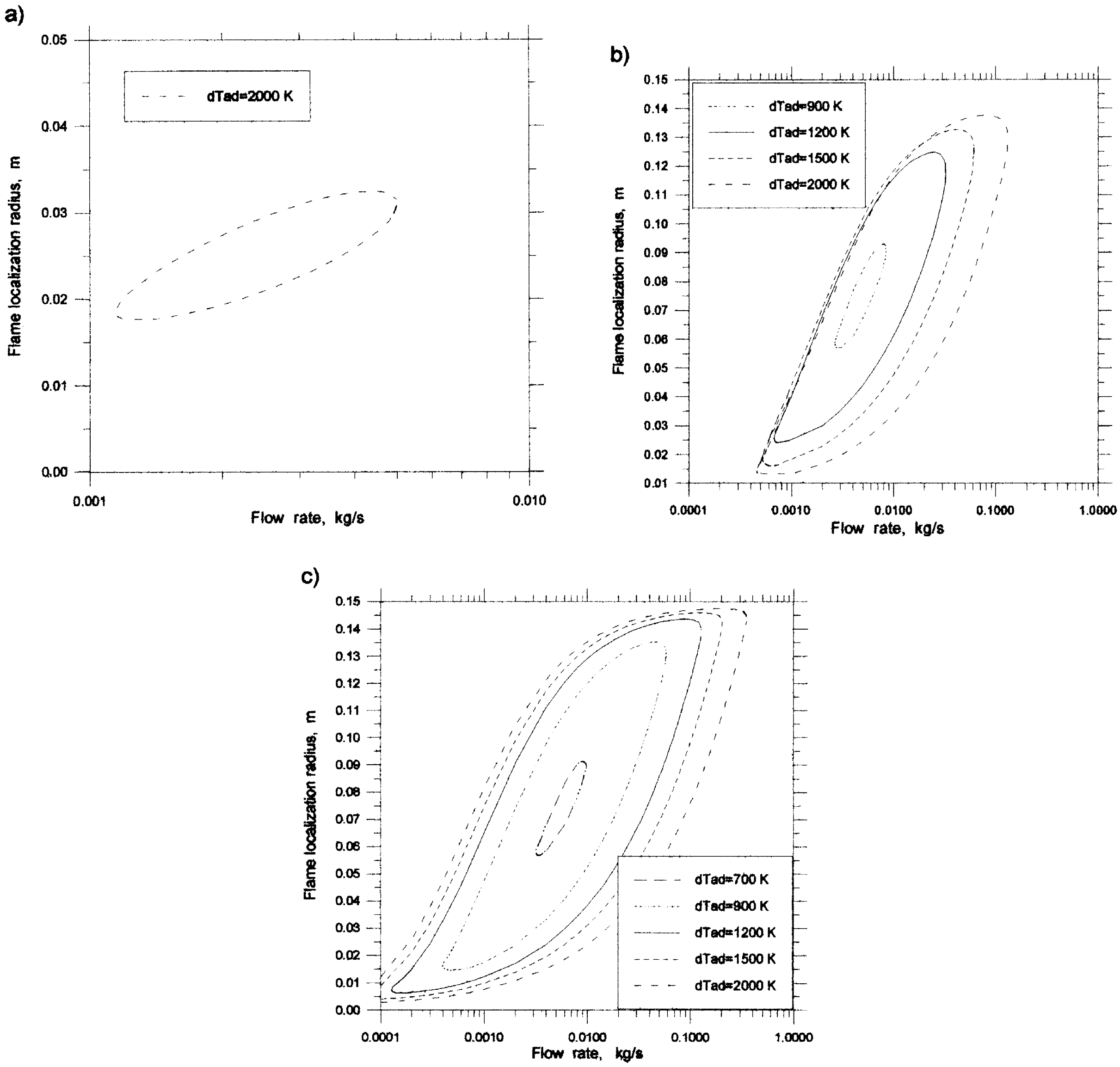

Fig. 4. Solution of the system (12), (16)–(17) for combustion front localization in spherical PMB. (a) Outer radius $r_2 = 0.05$ m, $d_0 = 5.6$ mm, (b) $r_2 = 0.15$ m, $d_0 = 5.6$ mm, (c) $r_2 = 0.15$ m, $d_0 = 1$ mm.

ization (close to blow-out limit in Figs 6 and 7) to such an extent that its position can be considered fixed at the initial ignition radius at times of the order of an hour. Geometrical length of such zones is defined by the system parameters. The plots also give the gas flow rate interval at which outside ignition of the filtration combustion is possible.

Similar computations were performed for spherical PMB. The results of numerical simulation are given in Figs 8 and 9.

5. Conclusion

Basic conclusions of the present study consist of the following:

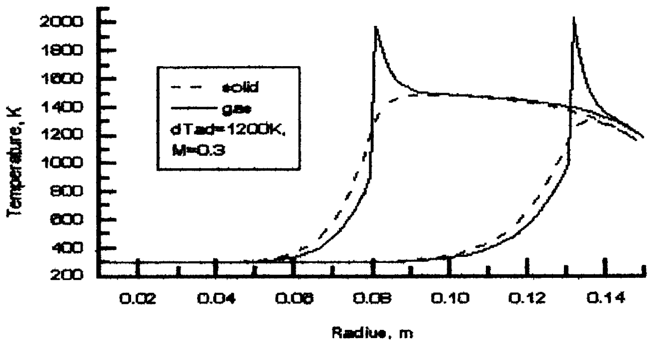

Fig. 5. Temperature profiles of filtration combustion wave, corresponding to two stationary localizations of the front. $\Delta T_{ad} = 1200$ K, $G_0 = 0.5$ kg/s, $r_2 = 15$ cm, $d_0 = 5.6$ mm.

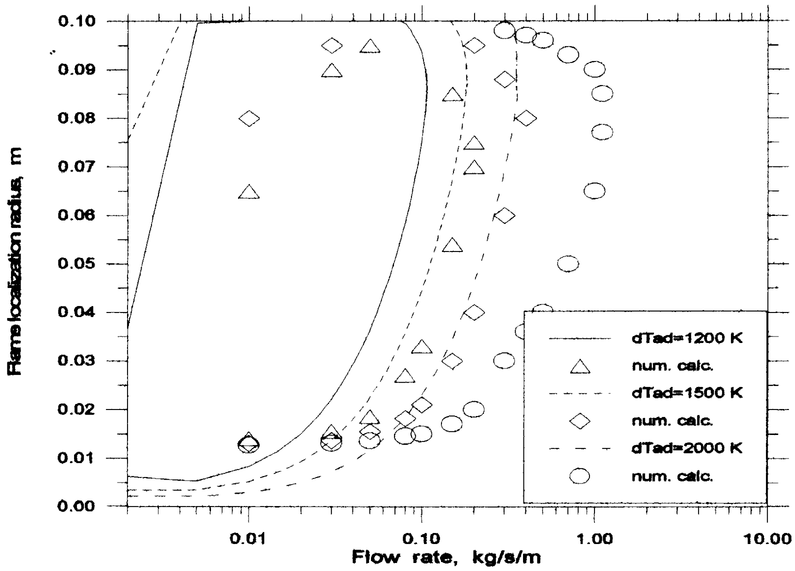

Fig. 6. Dependence of flame localization radii on flow rate. Numerical and analytical solution. Cylindrical burner outer radius $r_2 = 0.1$ m, $d_0 = 5.6$ mm, methane–air mixture.

(1) Combustion front localization in cylindrical and spherical PMB can depend on ignition radius.
(2) At high gas flow rate (close to combustion blow-out flow rate) 'indifferent' localization of combustion front takes place, i.e. a situation, when combustion front location is defined mainly by the coordinate of initial ignition of filtration combustion.
(3) Outside ignition of filtration combustion is possible for sufficiently calorific fuels and in a certain range of gas flow rates while ignition from inside is possible at wider range of parameters. Solution of the stated non-stationary problems (1)–(3), provides conditions for outside ignition of filtration combustion in the axis-symmetrical and spherical PMB ($\Delta T_{ad} \gtrsim 1500$ K for $G \simeq 0.03$–0.1 kg/(s m)).

The analysis performed in the article enables one to predict some effects related to the operation of filtration combustion burner of the considered geometry. Thus, for low-calorific fuels ($\Delta T_{ad} \approx 900$ K) combustion can occur only in a narrow area inside the carcass and at the limited range of fuel flow rates (Figs 3 and 4).

The present work did not consider the subject of combustion front stability, although it may be important for practical systems.

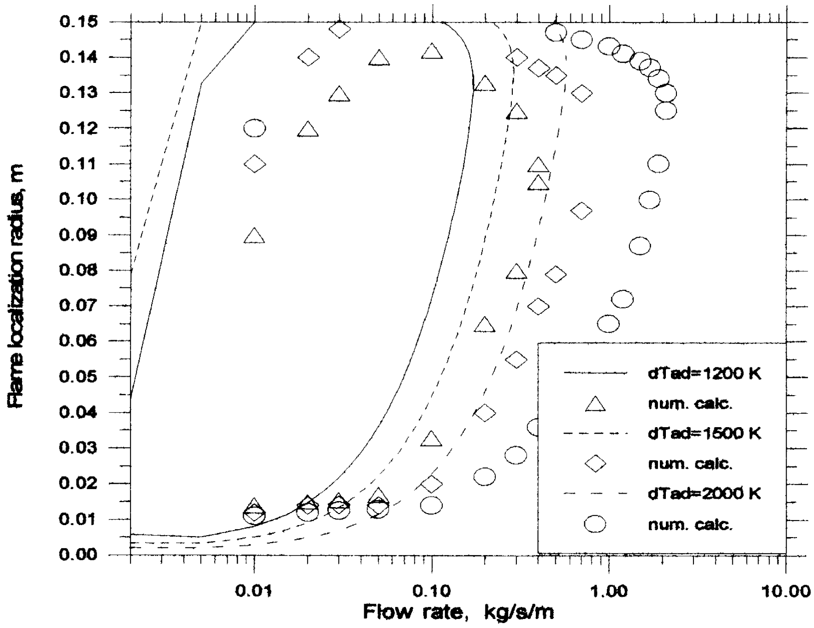

Fig. 7. Dependence of flame localization radii on flow rate. Numerical and analytical solution. Cylindrical burner outer radius $r_2 = 0.15$ m, $d_0 = 5.6$ mm, methane–air mixture.

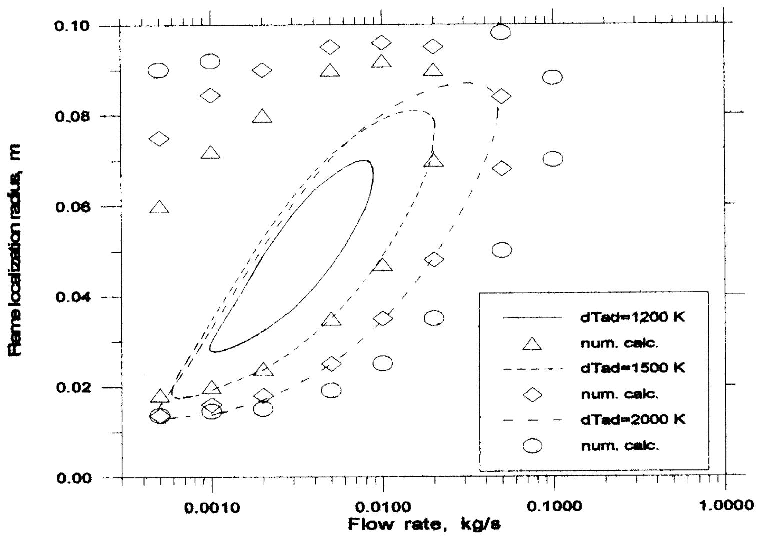

Fig. 8. Dependence of flame localization radii on flow rate. Numerical and analytical solution. Spherical burner outer radius $r_2 = 0.15$ m, $d_0 = 5.6$ mm, methane–air mixture.

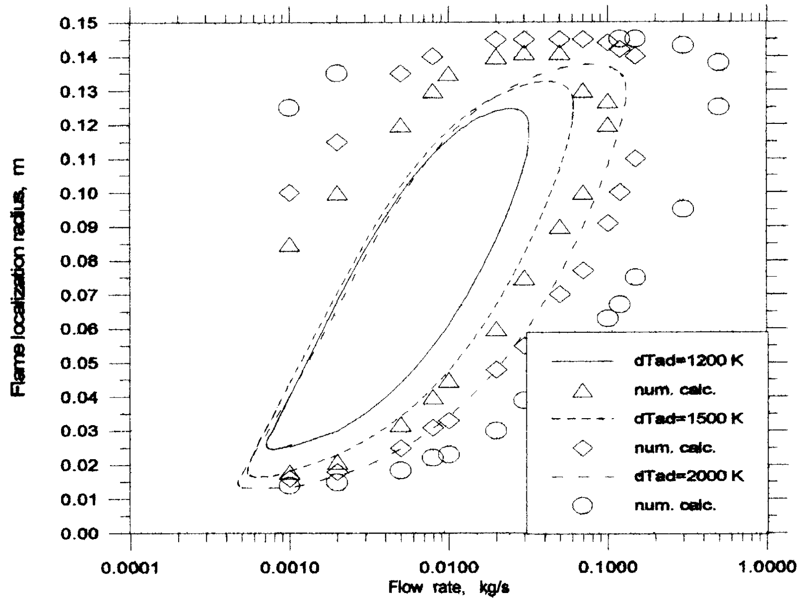

Fig. 9. Dependence of flame localization radii on flow rate. Numerical and analytical solution. Spherical burner outer radius $r_2 = 0.15$ m, $d_0 = 5.6$ mm, methane–air mixture.

References

[1] Weinberg FJ. Combustion temperature: the future? Nature 1971;233:239–41.

[2] Takeno T, Sato K. An excess enthalpy flame theory. Combustion Science and Technology 1979;20:73–84.

[3] Zhdanok S, Kennedy LA, Koester G. Superadiabatic combustion of methane–air mixtures under filtration in a packed bed. Combustion and Flame 1995;100:221–31.

[4] Laevski Yu M, Babkin VS. Filtration combustion of gases. In: Matros Yu, editor. Collected Book Propagation of Heat Waves in Heterogeneous Media. Novosibirsk, Hauka, 1988. pp. 108–45.

[5] Dobrego KV, Zhdanok SA. Engineering calculation of filtration combustion wave characteristics, based on two-temperature one-dimensional model. J. Engineering Physics and Thermal Physics 1998;71(3).

[6] Sahraoui M, Kaviani M. Direct simulation vs volume-averaged treatment of adiabatic, premixed flame in a porous medium. Int J Heat and Mass Transfer 1994;37:2817–34.

[7] Aerov ME, Todes OM, Narinskii DA. Apparati so stacionarnim zernistim sloem (in Russian). Chemia, 1979.

[8] Minaev SS, Potytnyakov SI, Babkin VS. Combustion wave instability in the filtration combustion of gases. Combustion, Explosion, and Shock Waves 1994;30(3):49–54.